

Spin magnetotransport in two-dimensional hole systems

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Spin current of two-dimensional holes occupying the ground-state subband in an asymmetric quantum well and interacting with static disorder potential is calculated in the presence of a weak magnetic field \mathbf{H} perpendicular to the well plane. Both spin-orbit coupling and Zeeman coupling are taken into account. It is shown that the applied electric field excites both the transverse (spin-Hall) and diagonal spin currents, the latter changes its sign at a finite H and becomes greater than the spin-Hall current as H increases. The effective spin-Hall conductivity introduced to describe the spin response in Hall bars is considerably enhanced by the magnetic field in the case of weak disorder and demonstrates a non-monotonic dependence on H .

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One of the most challenging problems in the physics of two-dimensional (2D) electron systems is the excitation of spin currents by an electric field \mathbf{E} directed in the 2D plane. This phenomenon exists owing to the spin-orbit (SO) interaction, which brings spin-dependent terms to the Hamiltonian of free electrons (intrinsic SO coupling) as well as spin-dependent corrections to the scattering potential (extrinsic SO coupling); see Ref. 1 for a review. In the quantum wells grown along [001] crystallographic direction in cubic crystals of zinc-blende type, the symmetry allows both non-diagonal (perpendicular to \mathbf{E}) and diagonal (parallel to \mathbf{E}) currents of z -polarized spins. The diagonal spin current does not appear if the SO splitting is isotropic. The presence of non-diagonal spin current leads to the spin Hall effect, which has been detected both in electron^{2,3} and hole^{4,5} systems by observing spin accumulation near the sample boundaries. The original theoretical proposal⁶ of the intrinsic spin Hall effect has been based on the Rashba Hamiltonian⁷ describing the SO coupling in electron systems due to structural inversion asymmetry. However, theoretical calculations⁸ have proved the absence of static intrinsic spin currents for this case, and this statement remains true when a magnetic field is applied to the system.⁹ Consideration of the equation of motion for the spin density operator¹⁰ allows one to extend applicability of this result of Refs. 8 and 9 to any electron system described by the \mathbf{p} -linear SO coupling. On the other hand, the static intrinsic spin currents are present in 2D hole systems described by the effective \mathbf{p} -cubic SO coupling Hamiltonian,¹¹ as demonstrated by theoretical studies.^{12–16} It is believed that the experimentally observed spin Hall effect for 2D holes^{4,5} is of the intrinsic origin.

In spite of the fact that experiments on spin excitation are often carried out in the presence of magnetic fields, the theoretical work devoted to the influence of a magnetic field on the electric-field-induced spin currents is very limited.¹⁷ For 2D hole systems, where the intrinsic spin currents exist, the spin-Hall conductivity has been calculated^{18,19} using Kubo formalism in the collisionless approximation. Although this approach allows one to study the case of strong magnetic field and to describe Shubnikov-de Haas oscillations of spin conductivity,¹⁹

it cannot be used for weak magnetic fields, when the cyclotron frequency is comparable to or less than the momentum relaxation rate due to scattering. In particular, the zero magnetic field limit of the spin-Hall conductivity appears to be singular in the collisionless approximation.¹⁸ To describe the region of weak magnetic fields, which is the most important for experimental studies, a consideration of spin conductivity in the presence of scattering is necessary.

In this Brief Report, the intrinsic spin current is calculated for the case of a classically weak magnetic field \mathbf{H} perpendicular to the 2D layer. The elastic scattering of carriers is taken into account. The general approach to the problem assuming arbitrary SO coupling Hamiltonian is followed by application to 2D hole systems described by the \mathbf{p} -cubic SO coupling. The analytical solution obtained below shows that the spin-Hall conductivity increases at small H and has a maximum in the region where the cyclotron frequency is smaller than the relaxation rate. Moreover, it is found that the diagonal component of the spin conductivity appears. For this reason, the spin response in Hall bars should be described by the effective spin-Hall conductivity which is a combination of diagonal and non-diagonal components of the spin conductivity tensor.

Consider 2D quasiparticles whose states are doubly degenerate in spin if the magnetic field and SO interaction are absent. In the presence of both the magnetic field and SO interaction, the single-particle Hamiltonian is

$$\hat{H} = \frac{\pi^2}{2m} + \hat{h}_{\pi} + V_{\mathbf{r}}, \quad \hat{h}_{\pi} = \hbar\Omega_{\pi} \cdot \hat{\sigma}, \quad (1)$$

where $\pi = -i\hbar\nabla_{\mathbf{r}} - (e/c)\mathbf{A}_{\mathbf{r}}$ is the operator of momentum in the 2D plane (xy) and $\mathbf{A}_{\mathbf{r}}$ is the vector potential describing the magnetic field according to $\mathbf{H} = [\nabla_{\mathbf{r}} \times \mathbf{A}_{\mathbf{r}}]$. Next, m and e are the effective mass and electric charge of the quasiparticles, c is the velocity of light, $V_{\mathbf{r}}$ is the scattering potential, and $\hat{\sigma}$ is the vector of Pauli matrices. The 2×2 matrix term \hat{h}_{π} describes both intrinsic SO coupling and Zeeman interaction. The extrinsic SO coupling effects are not considered.

The limit of weak (classical) magnetic field corresponds to the condition $\hbar\omega_c \ll \bar{\epsilon}$, where $\omega_c = |e|H/mc$ is the

cyclotron frequency and $\bar{\varepsilon}$ is the mean kinetic energy of quasiparticles. It is convenient to describe transport phenomena by using the kinetic equation for the Wigner distribution function $\hat{\rho}_{\text{prt}}$, which is a 2×2 matrix over the spin indices (see, for example, Refs. 20 and 21) and depends on the 2D momentum \mathbf{p} , coordinate \mathbf{r} , and time t . For the spatially-homogeneous and static case considered below, the coordinate and time dependence is omitted. Searching for the linear response to the applied electric field \mathbf{E} , one represents the distribution function in the form $\hat{\rho}_{\mathbf{p}} = \hat{f}_{\mathbf{p}}^{(eq)} + \hat{f}_{\mathbf{p}}$, where $\hat{f}_{\mathbf{p}}$ is the non-equilibrium part satisfying the linearized kinetic equation^{20,21}

$$\frac{i}{\hbar} [\hat{h}_{\mathbf{p}}, \hat{f}_{\mathbf{p}}] + \frac{e}{c} \left\{ [\hat{\mathbf{u}}(\mathbf{p}) \times \mathbf{H}], \frac{\partial \hat{f}_{\mathbf{p}}}{\partial \mathbf{p}} \right\} + e\mathbf{E} \cdot \frac{\partial \hat{f}_{\mathbf{p}}^{(eq)}}{\partial \mathbf{p}} = \hat{J}(\hat{f}_{\mathbf{p}}), \quad (2)$$

$\hat{\mathbf{u}}(\mathbf{p}) = \mathbf{v}_{\mathbf{p}} + \partial \hat{h}_{\mathbf{p}} / \partial \mathbf{p}$ is the group velocity in the presence of spin-orbit interaction, $\mathbf{v}_{\mathbf{p}} = \mathbf{p}/m$, and $\{\hat{a}, \hat{b}\} = (\hat{a}\hat{b} + \hat{b}\hat{a})/2$ denotes the symmetrized matrix product. The collision integral \hat{J} describing the elastic scattering is considered under the assumption that the relaxation rate is small in comparison to $\bar{\varepsilon}/\hbar$. Assuming that the spin-splitting energy $2\hbar|\Omega_{\mathbf{p}}|$ is small in comparison to $2\bar{\varepsilon}$, it is convenient to expand the collision integral in series of $\hbar|\Omega_{\mathbf{p}}|/\bar{\varepsilon}$. Retaining the first-order terms in this expansion, one obtains^{21,22}

$$\hat{J}(\hat{f}_{\mathbf{p}}) \simeq \frac{2\pi}{\hbar} \int \frac{d\mathbf{p}'}{(2\pi\hbar)^2} w_{|\mathbf{p}-\mathbf{p}'|} \left[(\hat{f}_{\mathbf{p}'} - \hat{f}_{\mathbf{p}}) \delta(\varepsilon_{\mathbf{p}'} - \varepsilon_{\mathbf{p}}) - \{(\hat{h}_{\mathbf{p}} - \hat{h}_{\mathbf{p}'}), (\hat{f}_{\mathbf{p}'} - \hat{f}_{\mathbf{p}})\} \frac{\partial \delta(\varepsilon_{\mathbf{p}'} - \varepsilon_{\mathbf{p}})}{\partial \varepsilon_{\mathbf{p}}} \right], \quad (3)$$

where $w_{|\mathbf{p}-\mathbf{p}'|}$ is the spatial Fourier transform of the correlation function of the scattering potential $V_{\mathbf{r}}$, and $\varepsilon_{\mathbf{p}} = p^2/2m$ is the kinetic energy. The condition $\hbar|\Omega_{\mathbf{p}}| \ll \bar{\varepsilon}$ also allows one to write the equilibrium distribution function in the form $\hat{f}_{\mathbf{p}}^{(eq)} \simeq f_{\varepsilon_{\mathbf{p}}} + \hat{h}_{\mathbf{p}} f'_{\varepsilon_{\mathbf{p}}}$, where $f'_{\varepsilon_{\mathbf{p}}} \equiv \partial f_{\varepsilon_{\mathbf{p}}} / \partial \varepsilon_{\mathbf{p}}$ and f_{ε} is the Fermi distribution function.

The calculations below are done in the approximation of short-range scattering potential, when $w_{|\mathbf{p}-\mathbf{p}'|} \simeq w$ is constant. It is convenient to use the spin-vector representation $\hat{f}_{\mathbf{p}} = \mathbf{f}_{\mathbf{p}}^0 + \hat{\sigma} \cdot \mathbf{f}_{\mathbf{p}}$. In the leading order with respect to the small parameter $\hbar|\Omega_{\mathbf{p}}|/\varepsilon_{\mathbf{p}}$ one finds

$$\mathbf{f}_{\mathbf{p}}^0 \simeq -G_{\mathbf{p}} f'_{\varepsilon_{\mathbf{p}}}, \quad G_{\mathbf{p}} = e \frac{\nu(\mathbf{E} \cdot \mathbf{v}_{\mathbf{p}}) - \omega_c [\mathbf{E} \times \mathbf{v}_{\mathbf{p}}] \cdot \mathbf{n}}{\nu^2 + \omega_c^2}, \quad (4)$$

where $\nu = mw/\hbar^3$ is the relaxation rate and $\mathbf{n} = (0, 0, 1)$ is the unit vector in the direction of \mathbf{H} . Here and below, e is taken to be positive, since the theory will be applied to hole systems. With the use of Eq. (4), the equation for the vector-function $\mathbf{f}_{\mathbf{p}}$ is written as

$$-2[\Omega_{\mathbf{p}} \times \mathbf{f}_{\mathbf{p}}] - \omega_c \frac{\partial}{\partial \varphi} \mathbf{f}_{\mathbf{p}} + \mathbf{R}_{\mathbf{p}} = \nu(\bar{\mathbf{f}}_{\mathbf{p}} - \mathbf{f}_{\mathbf{p}}), \quad (5)$$

where φ is the angle of the vector \mathbf{p} , and the line over a function denotes the angular average $(2\pi)^{-1} \int_0^{2\pi} d\varphi \dots$

The components of the vector-function $\mathbf{R}_{\mathbf{p}}$ are

$$R_{\mathbf{p}}^{\alpha} = \hbar\nu \frac{\partial}{\partial \varepsilon_{\mathbf{p}}} [(\Omega_{\mathbf{p}}^{\alpha} G_{\mathbf{p}} - \overline{\Omega_{\mathbf{p}}^{\alpha} G_{\mathbf{p}}}) f'_{\varepsilon_{\mathbf{p}}}] + \hbar e [\mathbf{E} \times \mathbf{n}] \cdot \frac{\partial \Omega_{\mathbf{p}}^{\alpha}}{\partial \varphi} \frac{f'_{\varepsilon_{\mathbf{p}}}}{p^2} - \hbar\omega_c \frac{\partial}{\partial \varphi} \left[\Omega_{\mathbf{p}}^{\alpha} \frac{\partial G_{\mathbf{p}} f'_{\varepsilon_{\mathbf{p}}}}{\partial \varepsilon_{\mathbf{p}}} \right]. \quad (6)$$

Equations (5) and (6) are valid for arbitrary $\Omega_{\mathbf{p}}$. The first and the second terms in Eq. (5) describe spin precession and cyclotron motion, respectively. The term $\mathbf{R}_{\mathbf{p}}$ describes excitation of spins by the electric field. Solution of Eq. (5) determines the non-equilibrium spin current density conventionally defined as

$$\mathbf{q}_{\gamma} = S \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} \text{Tr}(\{\hat{\sigma}, \hat{u}_{\gamma}(\mathbf{p})\} \hat{f}_{\mathbf{p}}), \quad (7)$$

where $S = 1/2$ for electrons and $S = 3/2$ for holes in the ground-state subband. Applying Eqs. (5)-(7) to electron system described by a \mathbf{p} -linear SO coupling Hamiltonian, one can find that the spin current is zero in the absence of Zeeman coupling, as expected.⁹ Let us consider the 2D holes near the bottom of the ground-state subband in an asymmetric [001]-grown quantum well. By adding the Zeeman term $-S\omega_H \hat{\sigma}_z$ to the effective \mathbf{p} -cubic SO coupling Hamiltonian¹¹ describing the 2D holes, one gets

$$\Omega_{\mathbf{p}} = (\kappa p^3 \cos 3\varphi, \kappa p^3 \sin 3\varphi, -3\omega_H/2), \quad (8)$$

where κ is a constant determined by the Luttinger parameters and confinement potential. Next, $\omega_H = g\mu_B H/\hbar$, where g is the effective g-factor of holes, and $\mu_B = |e|\hbar/2m_0c$ is the Bohr magneton. Expanding $\mathbf{f}_{\mathbf{p}}$ in series of angular harmonics, one can solve Eq. (5) exactly and obtain the result

$$\mathbf{q}_{\gamma} = (0, 0, q_{\gamma}), \quad q_{\gamma} = \Sigma_{\gamma\beta} E_{\beta}. \quad (9)$$

The tensor of spin conductivity, $\Sigma_{\gamma\beta}$, contains two contributions:

$$\Sigma_{\gamma\beta} = \Sigma_{\gamma\beta}^{(0)} + \Sigma_{\gamma\beta}^{(1)}, \quad (10)$$

$$\Sigma_{\gamma\beta}^{(0)} = \frac{9e\omega_H}{4\pi\hbar} \frac{\nu\delta_{\gamma\beta} + \omega_c e_{z\gamma\beta}}{\nu^2 + \omega_c^2} \int_0^{\infty} d\varepsilon_{\mathbf{p}} (-f'_{\varepsilon_{\mathbf{p}}}), \quad (11)$$

$$\Sigma_{\gamma\beta}^{(1)} = \frac{9e}{8\pi\hbar} \int_0^{\infty} d\varepsilon_{\mathbf{p}} (-f'_{\varepsilon_{\mathbf{p}}}) \text{Re} \frac{[i\delta_{\gamma\beta} + e_{z\gamma\beta}] \Delta_p^2 \nu \nu_-}{(\nu - i\omega_c)^2 (\nu_+ \nu_- + \Delta_p^2)}, \quad (12)$$

$$\nu_{\pm} = \nu - i\omega_c \pm 3i(\omega_c - \omega_H).$$

In these equations, $e_{z\gamma\beta}$ is the antisymmetric unit tensor and $\hbar\Delta_p = 2\hbar\kappa p^3$ is the SO splitting energy. Notice the symmetry properties $\Sigma_{xy} = -\Sigma_{yx}$ and $\Sigma_{xx} = \Sigma_{yy}$. The contribution $\Sigma_{\gamma\beta}^{(0)}$ exists owing to equilibrium spin polarization by the magnetic field in the presence of Zeeman coupling. Indeed, the equilibrium spin density is given

by $s_z = S \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} \text{Tr}(\hat{\sigma}_z \hat{f}_{\mathbf{p}}^{(eq)}) \simeq 9\omega_H m / 4\pi\hbar$, and the related spin conductivity is $\Sigma_{\gamma\beta}^{(0)} = (s_z / en_h) \sigma_{\gamma\beta}$, where $\sigma_{\gamma\beta}$ is the Drude conductivity tensor and n_h is the hole density. In contrast, the contribution $\Sigma_{\gamma\beta}^{(1)}$ is caused by the SO coupling. In zero magnetic field, when $\omega_c = \omega_H = 0$, $\Sigma_{\gamma\beta}^{(0)}$ disappears and $\Sigma_{\gamma\beta}^{(1)}$ contains only non-diagonal (Hall) component, $\Sigma_{xy}^{(1)} = 9e\Gamma / 8\pi\hbar$ (see Refs. 12-16), where the factor Γ describes suppression of the spin-Hall conductivity by the disorder. In the limit of low temperature, when the hole gas is degenerate, $\Gamma = [1 + (\nu/\Delta)^2]^{-1}$, where $\hbar\Delta \equiv \hbar\Delta_{p_F}$ is the SO splitting energy at the Fermi surface and $p_F = \sqrt{2m\varepsilon_F}$ is the Fermi momentum. The transition to the low-temperature limit in Eqs. (11) and (12) implies $-f'_{\varepsilon_p} = \delta(\varepsilon_p - \varepsilon_F)$, so the integral in Eq. (11) is equal to unity and the integration over energy in Eq. (12) is reduced to the substitution $\Delta_p \rightarrow \Delta$.

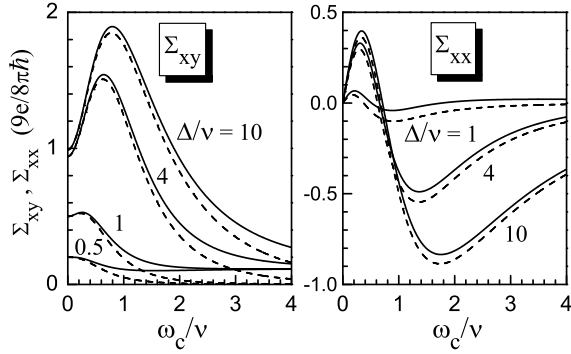


FIG. 1: The magnetic-field dependence of the spin conductivity components Σ_{xy} and Σ_{xx} (solid) and of their SO parts $\Sigma_{xy}^{(1)}$ and $\Sigma_{xx}^{(1)}$ (dash) for degenerate hole gas.

The main features of the spin conductivity in the magnetic field are the presence of both non-diagonal and diagonal components and the unusual (non-Drude) dependence of these components on the ratio ω_c/ν . Figure 1 shows the plots of Σ_{xy} and Σ_{xx} as functions of this ratio for several values of Δ/ν . Notice that the plots for $\Delta/\nu = 4$ correspond to experimental conditions of Ref. 5. The ratio $\omega_H/\omega_c = gm/2m_0$ is estimated as 0.06, using $g = 0.44$ and $m/m_0 \simeq 0.27$ given in Ref. 19. Since this ratio is small, the influence of Zeeman coupling on $\Sigma_{xy}^{(1)}$ and $\Sigma_{xx}^{(1)}$ is weak. However, $\Sigma_{xy}^{(1)}$ is considerably suppressed with the increase of ω_c , while $\Sigma_{xy}^{(0)}$ saturates at the value $(9e/8\pi\hbar)(gm/m_0)$ determined by the g-factor and effective mass. Therefore, the contribution $\Sigma_{xy}^{(0)}$ dominates in Σ_{xy} at larger ω_c/ν , especially for the dirty case $\Delta < \nu$, when the contribution $\Sigma_{xy}^{(1)}$ is suppressed. It is remarkable that Σ_{xy} is a non-monotonic function of the magnetic field and increases in the region of weak fields. This behavior takes place in the clean regime $\Delta \gg \nu$, when the spin splitting is not suppressed by the disorder. In the limit $\Delta \gg \nu, 3\omega_c$, the contribution $\Sigma_{xy}^{(1)}$ is

independent of the spin splitting:

$$\Sigma_{xy}^{(1)} \simeq \frac{9e}{8\pi\hbar} \frac{\nu^2}{\nu^2 + \omega_c^2} \left[1 + \frac{6\omega_c(\omega_c - \omega_H)}{\nu^2 + \omega_c^2} \right]. \quad (13)$$

According to this expression, the maximum $\Sigma_{xy}^{(1)}$ is approximately twice larger than the zero-field spin-Hall conductivity $9e/8\pi\hbar$. As follows from the Onsager symmetry principle, Σ_{xy} is symmetric with respect to the magnetic field reversal. In contrast, the diagonal component Σ_{xx} is antisymmetric in \mathbf{H} . This component is absent at $H = 0$ for the particular case of \mathbf{p} -cubic SO coupling considered here. As seen from Fig. 1, Σ_{xx} changes its sign at a finite magnetic field and is strongly suppressed by the disorder, though the magnetic-field suppression of this component is weak. The behavior of $\Sigma_{xx}^{(1)}$ in the limit $\Delta \gg \nu, 3\omega_c$ is described by the expression

$$\Sigma_{xx}^{(1)} \simeq \frac{9e}{8\pi\hbar} \frac{\nu\omega_c}{\nu^2 + \omega_c^2} \left[2 - 3\frac{\omega_H}{\omega_c} - \frac{6\omega_c(\omega_c - \omega_H)}{\nu^2 + \omega_c^2} \right]. \quad (14)$$

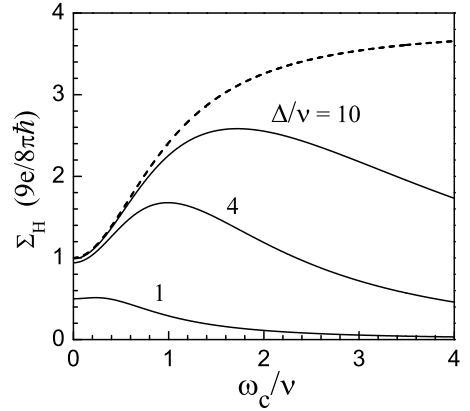


FIG. 2: The magnetic-field dependence of the effective spin-Hall conductivity Σ_H for the Hall bar geometry. The dashed line corresponds to the limit $\Delta \gg \nu, 3\omega_c$ given by Eq. (16).

In the experiments using the Hall bar geometry, the electric field acting on the carriers has both longitudinal and transverse (Hall) components, the latter is determined by the requirement of zero electric current in the transverse direction. If the field E_y is applied along the bar, the Hall field is $E_x = -(\sigma_{xy}/\sigma_{xx})E_y = -(\omega_c/\nu)E_y$, and the spin-Hall current $q_x = \Sigma_H E_y$ is described by the *effective* spin-Hall conductivity

$$\Sigma_H = \Sigma_{xy} - \frac{\omega_c}{\nu} \Sigma_{xx} = \frac{9e}{8\pi\hbar} \text{Re} \frac{\Delta^2 \nu_-}{(\nu - i\omega_c)(\nu + \nu_- + \Delta^2)} \quad (15)$$

written here for the case of degenerate hole gas. The Zeeman part $\Sigma_{\gamma\beta}^{(0)}$ does not contribute to Σ_H . The electric current through the Hall bar is $j_y = \sigma_0 E_y$, where $\sigma_0 = e^2 n_h / m\nu$ is the Drude conductivity at $H = 0$. Therefore, the quantity Σ_H/σ_0 determines the ratio of spin-Hall current to electric current in Hall bars. Figure 2 shows a considerable increase of Σ_H with respect to

its zero-field value and a non-monotonic behavior in the clean regime. If $\Delta \gg \nu, 3\omega_c$, one finds a simple expression

$$\Sigma_H \simeq \frac{9e}{8\pi\hbar} \left[1 + \frac{3\omega_c(\omega_c - \omega_H)}{\nu^2 + \omega_c^2} \right]. \quad (16)$$

The Hall bar also carries the longitudinal spin current $q_y = \Sigma_D E_y$ expressed through the effective diagonal spin conductivity $\Sigma_D = \Sigma_{xx} + (\omega_c/\nu)\Sigma_{xy}$.

Since the approach used in this Brief Report does not describe the Shubnikov-de Haas oscillations, the results presented above can be directly applied if these oscillations are suppressed either by the temperature or by the disorder. In the general case, Eqs. (10)-(16) should be treated as the expressions for the slow envelope part of the oscillating spin conductivity. If the relaxation rate ν is aimed to zero, $\Sigma_{xy}^{(1)}$ disappears at a finite magnetic field, in agreement with the result of Ref. 18. The effective spin-Hall conductivity Σ_H , however, remains finite in this limit.

To conclude, in the presence of a perpendicular magnetic field the spin current of 2D holes is described by the spin conductivity tensor containing both diagonal (Σ_{xx}) and non-diagonal (Σ_{xy}) components. These components are non-monotonic functions of H and increase in the low-field region. The enhancement of the spin-Hall component Σ_{xy} in comparison to its zero-field value and the

appearance of the diagonal component Σ_{xx} are explained by the asymmetry introduced by the Lorentz force, when the Fermi surface is shifted in the direction determined by the angle $\arctan(\omega_c/\nu)$ with respect to \mathbf{E} ; see Eq. (4). As the magnetic field increases and ω_c exceeds ν , the components Σ_{xx} and Σ_{xy} are suppressed by the field. The consideration of the component Σ_{xy} alone is not sufficient for description of the spin-Hall response in a Hall bar. It is necessary to introduce the effective spin-Hall conductivity Σ_H which determines the transverse spin current $q_x = \Sigma_H E_y$ proportional to the applied longitudinal electric field E_y . Since the spin density accumulation near the edges of the Hall bar is estimated as⁵ $s_z^{edge} \sim q_x m/p_F$, the behavior of Σ_H can be directly investigated by measuring the magnetic-field dependence of this accumulation. The theory suggests (see Fig. 2) that Σ_H is considerably enhanced by the magnetic field in the clean systems, where the spin coherence is not suppressed by the disorder. This condition is attainable in the existing samples^{4,5} and can be improved by increasing the hole density, because $\Delta \propto n_h^{3/2}$. If $\Delta/\nu \simeq 4$ (see Ref. 5), Σ_H has a maximum at $\omega_c \simeq \nu$, which corresponds to $H \simeq 2.8$ T if one uses the parameters $m = 0.27 m_0$ and $\hbar\nu = 1.2$ meV typical for 2D holes in GaAs quantum wells.^{4,5} Therefore, experimental verification of the theoretical results is possible and desirable.

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